

# CP-Violating Phases in the MSSM

T. Falk<sup>a \*</sup>

<sup>a</sup>School of Physics and Astronomy, University of Minnesota, Minneapolis, MN 55455, USA

We combine experimental bounds on the electric dipole moments of the neutron and electron with cosmological limits on the relic density of a gaugino-type LSP neutralino to constrain certain *CP*-violating phases appearing in the MSSM. We find that in the Constrained MSSM, the phase  $|\theta_\mu| \lesssim \pi/10$ , while the phase  $\theta_A$  remains essentially unconstrained.

The Minimal Supersymmetric Standard Model (MSSM) contains several new sources for *CP*-violation not present in the Standard Model, and it is well known [1–3] that these phases can produce large SUSY contributions to the electric dipole moments (EDM's) of the neutron and electron. The common generic description is that these contributions will exceed the current experiment limits on the neutron and electron EDM's [4,5] unless either the *CP*-violating phases are tiny ( $\theta < 0.01$ ) or the sfermion masses are very large ( $m_{\tilde{f}} > 1 \text{ TeV}$ ). However, large sfermion masses may be incompatible with bounds on the relic density  $\Omega_{\tilde{\chi}} h^2$  of a gaugino-type LSP neutralino. By combining cosmological and EDM constraints, we wish to find an upper bound on the magnitude of *CP*-violating phases within the MSSM.

In the MSSM, the Higgs mixing mass  $\mu$ , the gaugino mass parameter  $m_{1/2}$ , the scalar Higgs mixing parameter  $B\mu$ , and the trilinear couplings  $A$  are all potentially complex. However, not all of these phases are physical, and by rotating the gaugino and Higgs fields, one can eliminate the phases in all but  $\mu$  and the  $A$ 's [1].

The electric dipole moments of the quarks and electron receive SUSY contributions from the diagrams of Figure 1. Here  $\tilde{\lambda}$  can be either a gluino  $\tilde{g}$ , chargino  $\tilde{W}$  or neutralino  $\tilde{\chi}^0$ , and it is understood that an external photon line attaches to either the internal sfermion or  $\tilde{W}$  line. The necessary *CP*-violation either accompanies the mixing between

left and right-handed sfermions or arises from the mass/mixing matrices for the  $\tilde{W}$ 's or  $\tilde{\chi}^0$ 's (due to the presence of  $\mu$  in both mass matrices).

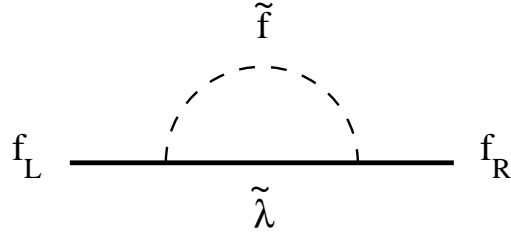


Figure 1. Diagrams contributing to quark and electron EDM's

Full expressions for the  $\tilde{W}$ ,  $\tilde{\chi}^0$  and  $\tilde{g}$  exchange contributions to the quark and electron EDM's in terms of the SUSY parameters can be found in [3]. The  $\tilde{g}$  exchange contribution to the quark EDM takes a particularly simple form:

$$d_q^g/e \sim \frac{\alpha_s}{\pi} \frac{m_q m_{\tilde{g}} |A^* + \mu \tan \beta|}{m_{\tilde{f}}^4} \sin \gamma \quad (1)$$

For up-type quarks, take  $\tan \beta \rightarrow \cot \beta$ . Here  $\gamma$  is the argument of the off-diagonal element of the squark mass matrix,  $\gamma = \arg(A^* + \mu \tan \beta)$ . For typical values of the masses,  $m_{\tilde{g}} = m_{\tilde{f}} = |A^* + \mu \tan \beta| = 100 \text{ GeV}$ , the requirement that the quark EDM contribution to the neutron EDM satisfy the experimental bound [4] of  $|d_n| < 1.1 \times 10^{-25} e \text{ cm}$  implies that the phase  $\gamma$  be very small,  $\sin \gamma \lesssim 0.001$ . However, this bound can be considerably relaxed by making the squarks heav-

\*To appear in the proceedings of SUSY-96: Supersymmetry and Unification of Fundamental Interactions, College Park, Maryland, USA 1996

ier. The  $\widetilde{W}$  exchange contribution also has a simple dependence on the SUSY phases, as  $d_q^c/e \sim \sin \theta_\mu$ , while the  $\tilde{\chi}^0$  exchange contribution has a more complicated dependence. Finally, we use the non-relativistic quark model to relate the neutron EDM to the up and down quark EDM's via  $d_n = (4d_d - d_u)/3$ .

We recall that a general neutralino is a linear combination of the neutral gauginos and higgsinos,  $\tilde{\chi}_i^0 = \alpha_i \widetilde{W}_3 + \beta_i \widetilde{B} + \gamma_i \widetilde{H}_1 + \delta_i \widetilde{H}_2$ . For large  $\mu > M_2, M_Z$ , however, the lightest neutralino is very pure bino. We consider the case of a bino as the lightest supersymmetric particle (LSP). To compute the  $\widetilde{B}$  relic density, we calculate the  $\widetilde{B}$  annihilation cross-section;  $\Omega_{\widetilde{B}} h^2 \sim \langle \sigma_{\text{ann}} v_{\text{rel}} \rangle^{-1}$ .  $\widetilde{B}$  annihilation is dominated by sfermion exchange into fermion pairs. This process exhibits “p-wave suppression”; that is, the zero-temperature annihilation rate is suppressed by powers of the final state fermion mass. Note that raising  $m_{\widetilde{f}}$  turns off this annihilation channel, and so bounding  $\Omega_{\widetilde{B}} h^2$  places an upper limit on the sfermion masses as well as on the  $\widetilde{B}$  mass.

It has been shown[6] that  $\Omega_{\widetilde{B}} h^2$  may in some cases be sensitive to the presence of  $CP$ -violating phases in the sfermion mass matrix. Since  $\widetilde{B}$ 's freeze out when they are non-relativistic, it is convenient to expand the annihilation cross-section  $\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle = a + b(T/m_{\widetilde{B}}) + \dots$ . In the absence of  $CP$ -violation and sfermion mixing, and taking  $m_{\widetilde{f}_1} = m_{\widetilde{f}_2}$ ,  $a$  is given by

$$a_f = \frac{g'^4}{128\pi} (Y_L^2 + Y_R^2) \frac{m_f^2}{(m_{\widetilde{f}}^2 + m_{\widetilde{B}}^2 - m_f^2)^2}, \quad (2)$$

and the p-wave suppression is evident, as  $a_f \sim m_f^2$ . Here  $Y_L(Y_R)$  is the left(right) sfermion hypercharge. In the presence of  $CP$ -violation and sfermion mixing, and taking  $m_{\widetilde{f}_1} \approx m_{\widetilde{f}_2}$ ,

$$a_f = \frac{g'^4}{32\pi} Y_L^2 Y_R^2 \frac{m_{\widetilde{B}}^2}{(m_{\widetilde{f}}^2 + m_{\widetilde{B}}^2 - m_f^2)^2} \times \sin^2 2\theta_f \sin^2 \gamma_f + O(m_f m_{\widetilde{B}}), \quad (3)$$

where  $\theta_f$  is the mixing angle between left and right sfermions, and  $\gamma_f$  is the phase described above. In this case,  $a_f$  contains a piece which is *not* p-wave suppressed.

In this talk, I will consider the case of the Constrained MSSM (CMSSM). In this Ansatz, the scalar masses are taken equal to a universal  $m_0$  at a unification scale  $M_X$ , the gaugino masses unify to  $m_{1/2}$ , and the trilinear couplings  $A_f$  are set equal to  $A_0$ . The renormalization group equations are then used to run the parameters down to the electroweak scale. We are left with two independent phases,  $\theta_\mu$  and  $\theta_A$  at the scale  $M_X$ .

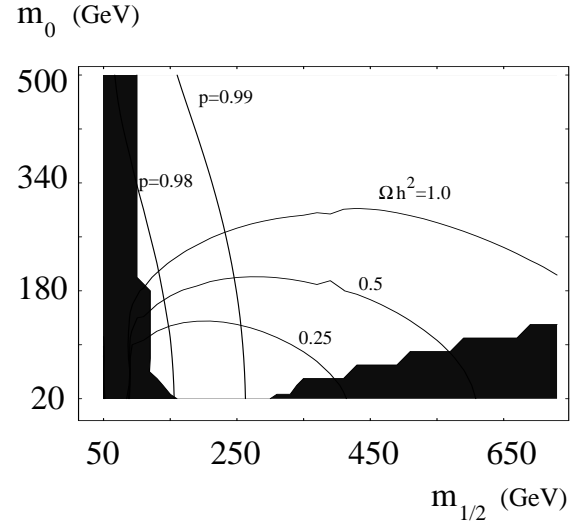


Figure 2.

In Figure 2 we display contours of constant  $\Omega_{\tilde{\chi}} h^2 = 0.25, 0.5$ , and  $1.0$ , as a function of  $m_0$  and  $m_{1/2}$ , for  $\tan \beta = 2.1$ , and taking  $A_0 = 300$  GeV,  $\theta_A = 0.8\pi$  and  $\theta_\mu = 0$  [7]. The dark regions are excluded because they produce either light  $\widetilde{W}$ 's or light sfermions or lead to staus or stops as the LSP. Also plotted are curves of constant  $\widetilde{B}$  purity, and we observe that the neutralinos near their mass upper bound for  $\Omega_{\tilde{\chi}} h^2 = 0.25$  are very pure ( $p > 0.99$ ) bino, so that the  $\tilde{\chi}^0$ 's will annihilate predominantly through sfermion exchange, as described above. Requiring  $\Omega_{\tilde{\chi}} h^2 \leq 0.25$ , the resulting upper bound on  $m_{1/2}$  is  $\approx 400$  GeV, corresponding to  $m_{\widetilde{B}} \lesssim 160$  GeV.

This bound is quite independent of the parameters  $A_0, \theta_A$  and  $\theta_\mu$ . Recall that the lifting of the p-wave suppression described above re-

quires both  $CP$ -violation and significant sfermion mixing (though it can be lifted to some extent by sfermion mixing alone). Since annihilation into leptons is particularly enhanced, we consider stau mixing. At the electroweak scale, the left and right stau mass parameters are split by  $m_L^2 - m_R^2 \approx 0.4m_{1/2}^2$ . Then for the part of the zero temperature cross-section which is not p-wave suppressed,  $\langle\sigma_{\text{ann}}v_{\text{rel}}\rangle_{T=0} \sim \sin^2 2\theta_\tau$ , where

$$\sin^2 2\theta_\tau \approx 0.01 \left( \frac{100 \text{ GeV}}{m_{1/2}} \right)^4 \left( \frac{A^* + \mu \tan \beta}{100 \text{ GeV}} \right)^2 \quad (4)$$

This is very small for  $m_{1/2}$  near its upper bound, and so the zero temperature annihilation rate remains suppressed.

The neutron EDM is sensitive to the masses of the squarks, which in the CMSSM are given by  $m_q^2 \approx m_0^2 + 6m_{1/2}^2 + O(m_Z^2)$ . These masses, and consequently the neutron EDM, are insensitive to  $m_0$  in the cosmologically allowed region (see Figure 2). We also find that the dominant contribution to the neutron EDM comes from  $\tilde{W}$ -exchange (unless  $\theta_A \gg \theta_\mu$ ), and so is insensitive to  $A_0$ . Minimizing the neutron EDM is then achieved by taking  $m_{1/2}$  as large as is cosmologically allowed.

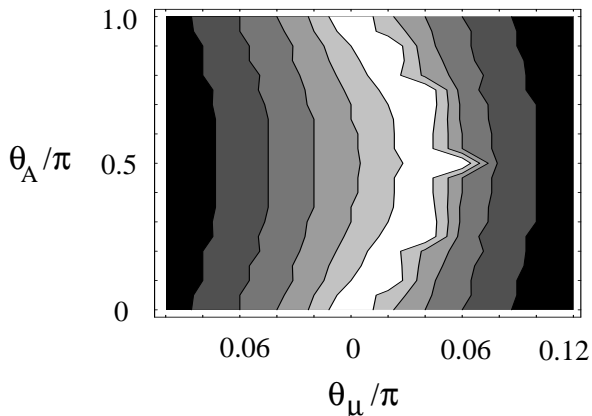


Figure 3. Minimum value of  $m_{1/2}$  needed to bring the neutron EDM below experimental bounds.

Accordingly, in Figure 3 we plot, as a function of  $\theta_\mu$  and  $\theta_A$ , the *minimum* value of  $m_{1/2}$  needed to bring the neutron EDM down below its experimental bound of  $1.1 \times 10^{-25} e \text{ cm}$ . The

light central region has  $m_{1/2}^{\text{min}} < 200 \text{ GeV}$ , and successive contours represent steps of 100 GeV. The black regions yield a stau as the LSP. Since  $\tilde{W}$  exchange dominates unless  $\theta_A \gg \theta_\mu$ , and since  $\theta_A$  contributes only part of  $\gamma$ , the neutron EDM is fairly insensitive to  $\theta_A$ . The contours are bowed to the right of  $\theta_\mu = 0$ , where there is a cancellation between the  $\tilde{W}$  and  $\tilde{g}$  exchange contributions. There are also similar allowed regions near  $\theta_\mu = \pi$  and for negative  $\theta_A$ . Recalling that  $\Omega_{\tilde{\chi}} h^2 < 0.25$  requires  $m_{1/2} \lesssim 400 \text{ GeV}$ , we see from Figure 3 that  $|\theta_\mu| \lesssim \pi/10$ , while  $\theta_A$  is essentially unconstrained.

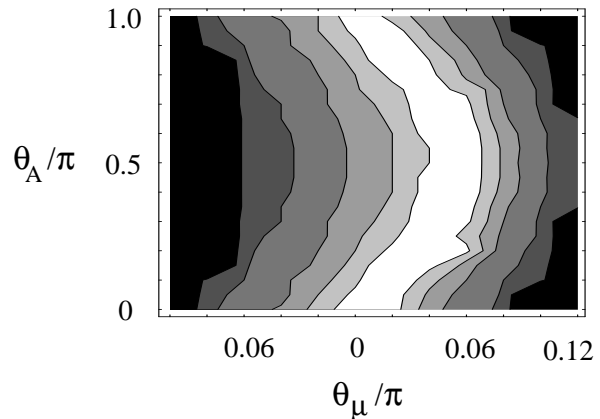


Figure 4. Same as Figure 3 for the  $e^-$  EDM

The above bounds may be sensitive to the spin structure of the nucleon[8], so it is important to also consider bounds from the electron EDM. In Figure 4, we require  $m_{1/2}$  to be large enough so that the  $e^-$  EDM is less than  $1.9 \times 10^{-26} e \text{ cm}$ [5]. We find the bounds on  $\theta_\mu$  from the  $e^-$  EDM are comparable to those from the neutron EDM.

I would like to gratefully acknowledge Keith Olive and Mark Srednicki, with whom this work was done in collaboration.

## REFERENCES

1. M. Dugan, B. Grinstein & L. Hall, Nucl. Phys. **B255** (1985) 413.
2. J. Ellis, S. Ferrara, and D.V. Nanopoulos, Phys. Lett. **114B** (1982) 231; W. Buchmüller

- and D. Wyler, Phys. Lett. **121B** (1983) 321;  
J. Polchinski and M. Wise, Phys. Lett. **125B**  
(1983) 393; F. del Aguila, M. Gavela, J. Gri-  
fols, and A. Mendez, Phys. Lett. **126B** (1983)  
71; D.V. Nanopoulos and M. Srednicki, Phys.  
Lett. **128B** (1983) 61.
3. Y. Kizukuri & N. Oshimo, Phys. Rev. **D45**  
(1992) 1806; **D46**
  4. I. S. Altarev *et al.*, Phys. Lett **B276** (1992)  
242.
  5. Abdullah *et al.*, Phys. Rev. Lett. **65**, (1990)  
2347.
  6. For details, see T. Falk, K.A. Olive and M.  
Srednicki, Phys. Lett. **B354**, (1995) 99.
  7. For details, see T. Falk and K.A. Olive, UMN-  
TH-1423-96, hep-ph/9602299.
  8. J. Ellis and R. Flores, hep-ph/9602211  
(1996).